M480: Introductory Activity

The following questions provide a brief glimpse into the world of combinatorics. Since we have not yet provided any theorems/rules, you are encouraged to work through these problems with a partner and try to give the final answer. You should also write down any processes you used to get the final answer. If you have answered one of these questions before, you should skip it for now.

Count 1. In a "round-robin" tournament, each team plays every other team exactly once. How many total games are played if there are 100 teams?

Count 2. A standard Connect-Four board is a 6×7 board that stands vertically (see Figure 1). How many ways can you populate the board using only red discs?[1]

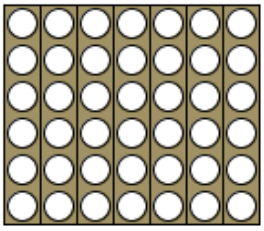

Figure 1: A picture of a Connect-Four board.

[1] In a usual game, players alternate playing red discs and black discs. Using this restriction, the problem becomes a bit harder, but you can think about that one too.

Count 3. There are six students in a class and they each hand in a homework assignment. How many ways can I distribute the assignments to the students (so they can grade them) with the restriction that no student gets his or her own assignment?[2]

[2] It is not fair to grade your own assignment.

Count 4. An icosahedron is a regular solid made from 20 identical equilateral triangles (see Figure 2). If we can paint each triangle red, blue, green, or yellow, how many different ways are there to paint the entire icosahedron?

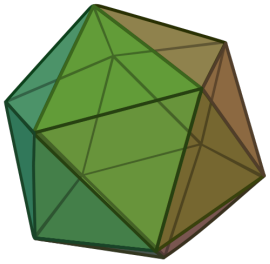

Figure 2: An icosahedron.

Count 5. Suppose you have a 60×60 checkerboard. On the upper left square, a banker places a penny. In the next square to the right, she places 2 pennies, and the next she places 8 pennies. Suppose she continues this process (doubling the pennies on each successive square and moving to the next row like a typewriter).

1. How much money will be in the first row?
2. How much money will be on the entire board?
3. How much money will be in the first column?

Count 6. In the game of Black-Jack, you are initially dealt two cards: one face-down and the other face-up. How many ways can you be dealt an Ace face-down and a heart face-up? How about an Ace face-down and the face-up card is *not* a heart?

Count 7. Of the integers from 1 to 999999, how many are palindromes?[3]

[3] A *palindrome* is a number the reads the same forwards or backwards, such as 34543.