Name:_____

M433: Abstract Algebra Activity

1. Write down the first few things that come to your mind when you think about "algebra." What do you think algebra is? What has your experience with algebra been?

2. Using the two different "multiplication" (operation) tables of "hatted numbers" and greek letters given below answer the following questions.

$\cdot$	$\hat{0}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$
$\hat{0}$	$\hat{0}$	$\hat{0}$	$\hat{0}$	$\hat{0}$	$\hat{0}$	$\hat{0}$	$\hat{0}$	$\hat{0}$
$\hat{1}$	$\hat{0}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$
$\hat{2}$	$\hat{0}$	$\hat{2}$	$\hat{4}$	$\hat{6}$	$\hat{0}$	$\hat{2}$	$\hat{4}$	$\hat{6}$
$\hat{3}$	$\hat{0}$	$\hat{3}$	$\hat{6}$	$\hat{1}$	$\hat{4}$	$\hat{7}$	$\hat{2}$	$\hat{5}$
$\hat{4}$	$\hat{0}$	$\hat{4}$	$\hat{0}$	$\hat{4}$	$\hat{0}$	$\hat{4}$	$\hat{0}$	$\hat{4}$
$\hat{5}$	$\hat{0}$	$\hat{5}$	$\hat{2}$	$\hat{7}$	$\hat{4}$	$\hat{1}$	$\hat{6}$	$\hat{3}$
$\hat{6}$	$\hat{0}$	$\hat{6}$	$\hat{4}$	$\hat{2}$	$\hat{0}$	$\hat{6}$	$\hat{4}$	$\hat{2}$
$\hat{7}$	$\hat{0}$	$\hat{7}$	$\hat{6}$	$\hat{5}$	$\hat{4}$	$\hat{3}$	$\hat{2}$	$\hat{1}$

$*$	α	β	γ	δ
α	β	γ	γ	δ
β	γ	β	α	β
γ	δ	γ	δ	γ
δ	γ	β	δ	α

   (a) What is $\hat{7} \cdot \hat{4}$? What is $\hat{4} \cdot \hat{7}$?

   (b) Can you find a solution to the equation $\hat{5} \cdot x = \hat{3}$? What about $\hat{5} \cdot x = \hat{2}$?

   (c) Find all the solution(s) to the equation $\hat{2} \cdot x = \hat{4}$.

   (d) Is there a "hatted number" that represents $\frac{\hat{1}}{\hat{5}}$? Why or why not?

   (e) Is there a "hatted number" that represents $\frac{\hat{1}}{\hat{4}}$? Why or why not?

   (f) Suppose that $x \cdot y = \hat{0}$. Does this mean that $x = \hat{0}$ or $y = \hat{0}$?

   (g) What is $\alpha * \beta$? What is $\beta * \alpha$?

   (h) Can you find a solution to the equation $\gamma * x = \beta$? What about $\gamma * x = \delta$?

   (i) Is there a greek letter that represents $\frac{\alpha}{\beta}$? Why or why not?

   (j) Can you use the tables to figure out what is meant by $\hat{4} \cdot \hat{5} \cdot \hat{6}$? Explain.

   (k) Can you use the tables to figure out what is meant by $\alpha * \beta * \gamma$? Explain.

Your group should have a polygon.

3. What kind of polygon do you have?

4. How many ways are there to place your polygon into the provided template? Describe all the positions using the numbers on the polygon.

5. Describe the "moves" you perform on the polygon to take it from its original position to each of the positions you have from above.

6. Is there a minimal way to describe the "moves" to someone? How many "basic moves" would you have to describe? Are there "moves" that are equivalent to others (meaning they are different moves but leave the polygon in the same ending position)? Think hard, are there different minimal types of "basic moves?"

7. Can you describe what happens if you perform two consecutive "moves"?

8. If you perform two "moves," does the order in which you perform them matter?

9. Find a convenient/compact way to describe the set of all the symmetries of your n-gon.

10. To think about: Based on your data, what do you think happened for other groups with different n-gons?

11. To think about: Can you do the same kind of analysis for a three-dimensional polyhedron (such as a cube)?