

## Setting the Stage The Box Problem

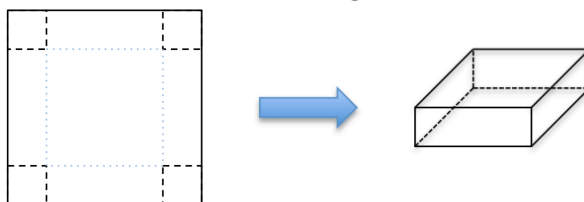
Here is an activity that we can use at the beginning of a calculus class that can be used to accomplish the following:

- Establish a culture of collaboration, including norms
- Establish the role of functions in our world and the many ways that we can analyze them without calculus
- Establish the fact that we need to do the work – just seeing a professor solve a problem is not enough.

The problem itself is not flashy nor of much importance other than it serves as a review of many of the mathematics skills and procedures that students have seen before calculus. Moreover, it establishes the reasons why we must focus on learning and not just reproducing the material. Finally, we will first present how I facilitated the activity the first time I tried it and then add an element that made the facilitation much more effective – group roles. Now for the problem itself.

**Example 1.** You are asked to help your company design a box with maximum volume given the following constraints:

- The box must be made from the following material – an 8.5” by 8.5” piece of cardboard.
- To create the box, you are asked to cut the same size square from each corner of the 8.5” by 8.5” piece of cardboard and to fold the remaining cardboard as in Figure 1.



**Figure 1.** Creating box from 8.5” by 8.5” piece of cardboard.

Most of us will recognize this problem as one found in most standard calculus textbooks. The difference is that we assign this problem without showing them how we would solve the problem. There are many reasons for doing this – it shows students the value of persistence, it establishes a problem solving culture in the classroom, it is review of much of the mathematics they have seen up to this point. Moreover this problem helps students begin to understand why functions are important and the different ways that we can analyze this function.

The first time I used this activity I began by posing the problem and letting them struggle for 5-10 minutes, always reading the students for their level of frustration. Most of the time students ask for steps and I offer hints when I think it is appropriate. Since students have different levels of understanding it can take a while for them to gain any traction. I brought scissors and sheets of paper and offered these materials to students. Some of them took these materials and some of them didn't. As well as the activity worked the first time not every student was able to come away with the main goals of the lesson.

This past year I made modifications in the facilitation of the activity that improved the desired outcomes. First, we discussed norms for working in groups. Most of our students attended high

schools where collaborative work is the norm so they were able to develop norms quickly. I then placed the students into groups of four, gave them colored name tags (Yellow, Green, Red, and Blue) had them introduce themselves to each other and discuss the following question:

“What are functions and why are they important?”

Before I asked them to share their answers I assigned the following roles that matched the color of their name tags:

- **Interrogator** (Yellow)  
The only person in the group who is allowed to ask me questions during this activity.
- **Investigator** (Green)  
At certain times, I will allow you to visit other groups to investigate what they're doing.
- **Reporter** (Red)  
Responsible for explaining your group's thinking to the whole class at certain times. Also responsible for uploading solution to Titanium.
- **Crew** (Blue)  
Responsible for handling tools for solving the task. You may be given special tools during the activity.

I then asked the Reporter to take out a cell phone, tablet, or computer and I used Poll Everywhere ([www.polleverywhere.com](http://www.polleverywhere.com)) to gather their responses to the question above. I showed the class the answers but I did not let them know if they were correct or not. Afterwards I asked each Crew member to pick up a worksheet with the task (see attached) and begin the activity. As I walked around I made sure to only talk to the interrogators and Reporters, either answering questions or asking for progress. It was nice to see the Interrogators or the Reporters confer with their groups as I stopped at each group – conversations were happening. After 10 minutes I noticed some groups were starting to struggle so I called the Crew Members to pick up some materials (scissors and paper) and suggested to the Crew that they may want to construct models to help them understand the problem. At this point many groups diverged in their approach to the problem. Some groups were constructing many models and finding the volumes, other only constructed one and started developing the function  $V(x) = x(8.5 - 2x)^2$ , whereas others did not touch the manipulatives and went straight to the function. It is at this point that I called the Investigators to me and I went over their role one more time. As soon as the investigators spread out the room exploded with productive conversations and it was nice to see different students see different approaches to solve a problem. They were then able to go back to each of the groups and quickly each group had the desired function.

It is at this point where students once again diverged in their approach to their problem. Some students started to plug in numbers into the function, some started to make tables, others tried to graph the function. A couple of groups asked me if they could use their graphing calculators. Of course I said yes and it was nice to see different approaches to this “problem.” At the end of this activity I had each Reporter discuss their approach to the problem and the “solution” that they found. I then reposed my original question on Poll Everywhere

“What are functions and why are they important?”

The answers the second time were much better and I was able to use this activity to not only review functions but establish our norms for collaborative work for the rest of the semester.