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★**Semigroups of linear operators and applications to partial differential equations.**

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Let L be a linear partial differential operator acting on functions of $x=(x_1,x_2,\cdots,x_m)\in\mathbf{R}^m$. The initial value problem (1) $u_t(x,t)=Lu(x,t)$ $(t\geq 0)$, $u(x,0)=u_0(x)$, in a domain $\Omega\subset\mathbf{R}^m$ (with boundary conditions if $\Omega\neq\mathbf{R}^m$) can be viewed as an initial value problem for an ordinary differential equation (in an infinite-dimensional space) as follows. Denote by A the operator L acting in a suitable space E of functions $u(x)$ defined in Ω, the boundary conditions (if any) included in the definition of the space or of the domain $D(A)$. Then (1) will be equivalent to (2) $u'(t)=Au(t)$ $(t\geq 0)$, $u(0)=u_0$. Assuming that solutions of (2) exist for a dense set of initial data u_0 and that $u(t)$ depends continuously on $u(0)$ (uniformly on compacta of $t\geq 0$) we can define a strongly continuous solution operator $S(t)$ of (2) by $S(t)u(0)=u(t)$ which will satisfy the semigroup or exponential equations (3) $S(0)=I$, $S(s+t)=S(s)S(t)$. Conversely, every strongly continuous operator-valued function $S(t)$ satisfying (3) is the solution operator of an abstract differential equation (2), with $Au=S'(0)u$, $D(A)$ consisting of all u for which the derivative exists. This way of looking at (1) and other equations was initiated independently almost four decades ago by K. Yosida and E. Hille ; it is now called semigroup theory and has proven to be very successful not only in handling classical initial-boundary value problems like (1) but also in areas such as functional differential equations and control theory. Frequently, the abstract formulation is not only a mathematical convenience but it is physically significant; for instance, in Maxwell's equations E is an L^2 space whose norm is (the square root of) the energy of the electromagnetic field, and in diffusion equations one may use as E an L^1 space whose norm measures the amount of diffusing matter.

Semigroup theory has not lacked textbooks, beginning with E. Hille [Functional analysis and semigroups, Amer. Math. Soc., New York, 1948; MR0025077], rewritten in 1957 with R. S. Phillips [Amer. Math. Soc., Providence, R.I., 1957; MR0089373]; a more recent monograph is by S. G. Krein [Linear differential equations in Banach spaces (Russian), "Nauka", Moscow, 1967; MR0247239; English translation, Amer. Math. Soc., Providence, R.I., 1971; MR0342804]. However, the applications (especially to partial differential equations) were not covered in detail in any of these, and one had to look them up in research papers or in partial differential equations textbooks such as the one by A. Friedman [Partial differential equations, Holt, Rinehart and Winston, New York, 1969; MR0445088], where the emphasis lies elsewhere. Hence the need for a monograph with reasonable depth to cover what is essential to the theory, as well as the necessary groundwork for nontrivial applications. This book (which is a corrected and expanded version of a widely circulated set of lecture notes that appeared in 1974) addresses that need in a very effective way. Chapter 1 treats the basic Hille-Yosida generation theorem as well as the particular case where $\|S(t)\|\leq 1$ with its associated theory of dissipative operators in Banach spaces. Chapter 2 deals, among other things, with analytic semigroups and semigroups of compact operators. Chapter 3 is on perturbation of generators and approximation of semigroups both in the continuous and in the discrete case. Chapter 4 studies the relation of the abstract differential equation (2) with strongly continuous semigroups.

During the 1960s and 1970s, semigroup theory extended in at least three (partially overlapping) directions. The first deals with distribution solutions of (2), the second with abstract time-dependent equations $u'(t) = A(t)u(t)$ and the third with nonlinear abstract differential equations. The last two are covered in the book in Chapters 5 and 6, respectively; the nonlinear equations here are mostly quasilinear, i.e., nonlinear perturbations of (2) of the form $u'(t) = Au(t) + f(t; u(t))$. Finally, Chapters 7 and 8 are on applications to partial differential equations, both linear and nonlinear.

The few prerequisites needed and the clear exposition make this monograph an excellent textbook for a graduate course or for part of a functional analysis or a partial differential equations sequence (although the lack of exercises does not help here). It presents a reasonably complete and deep overview of an important theory and of some of its applications; thus it will be of interest to users of partial differential and other equations. Finally, it offers the tools necessary for access to areas of mathematics where important research is presently going on, chiefly the theory of nonlinear semigroups. On these and other counts, it is a welcome addition to the literature. *H. O. Fattorini*