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Some new q-congruences for truncated basic hypergeometric series: even powers. (English summary)

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In this note, the authors establish several new q-congruences for truncated basic hypergeometric series with the base being an even power of q. For example, they prove the following results.

Theorem 1. Let n be an odd integer greater than 1. Then

$$\sum_{k=0}^{n-1}[4k+1]\frac{(q;q^2)_k^2}{(q^2;q^2)_k^2}q^{-k}\equiv q[n]^2 \pmod{[n]^2\Phi_n(q)}, \tag{1}$$

$$\sum_{k=0}^{n-1}[4k-1]\frac{(q^{-1};q^2)_k^2}{(q^2;q^2)_k^2}q^{k}\equiv -[n]^2 \pmod{[n]^2\Phi_n(q)}. \tag{2}$$

Theorem 2. Let $d\geqslant 4$ be an even integer and let n be a positive integer with $n\equiv -1 \pmod{d}$. Then

$$\sum_{k=0}^{n-1}[2dk+1]\frac{(q;q^d)_k^d}{(q^d;q^d)_k^d}q^{\frac{d(d-3)k}{2}}\equiv 0 \pmod{\Phi_n(q)^2}. \tag{3}$$

Theorem 3. Let $d\geqslant 4$ be an even integer and let $n>1$ be an integer with $n\equiv 1 \pmod{d}$. Then

$$\sum_{k=0}^{n-1}[2dk-1]\frac{(q^{-1};q^d)_k^d}{(q^d;q^d)_k^d}q^{\frac{d(d-1)k}{2}}\equiv 0 \pmod{\Phi_n(q)^2}. \tag{4}$$

Theorem 4. Let $d\geqslant 4$ be an integer and let n be a positive integer with $n\equiv -1 \pmod{d}$. Then

$$\sum_{k=0}^{n-1}[2dk+1]\frac{(q;q^d)_k^d}{(q^d;q^d)_k^d}q^{\frac{d(d-3)k}{2}}\equiv 0 \pmod{\Phi_n(q)^2\Phi_{dn-n}(q)}. \tag{5}$$

The authors also pose some conjectures in this paper. *Chen Wang*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.