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Mayerhofer, Eberhard (IRL-UL)
On Wishart and noncentral Wishart distributions on symmetric cones. (English summary)
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A noncentral Wishart distribution $\Gamma(\beta,\Sigma,\Omega)$ is concentrated on the cone S_m^+ of the positive semidefinite real matrices of order m and is defined by its Laplace transform

$$u \mapsto \det(I_m+2u)^{-\beta}\exp\left(-2\operatorname{tr}(\Omega\, u\Sigma(I_m+2u\Sigma)^{-1})\right).$$

Here $\beta>0$, Ω is in S_m^+ and Σ is positive definite of order m. The paper proves that $\Gamma(\beta,\Sigma,\Omega)$ exists if and only if the following two conditions hold: the Gindikin condition,

$$\beta\in\left\{0,\frac{1}{2},\frac{2}{2},\dots,\frac{m-1}{2}\right\}\cup\left(\frac{m-1}{2},\infty\right),$$

and the Mayerhofer condition, $\frac{1}{2}\operatorname{rank}\Omega\le\beta$. This paper gives the third, and the most elegant, proof of this difficult result, which has been known as the Mayerhofer conjecture after its formulation in 2011. The Graczyk, Małecki and Mayerhofer proof (2017) uses a special stochastic process on S_m^+, the Letac and Massam proof (2018) uses the zonal polynomials for describing $\Gamma(\beta,\Sigma,\Omega)$, and the present proof reactivates an idea of Peddada and Richards (1991) by a deeper analysis of the positivity of the generalized binomial coefficients occurring in a Euclidean Jordan algebra, as studied in the book of J. Faraut and A. Korányi [*Analysis on symmetric cones*, Oxford Math. Monogr., Oxford Univ. Press, New York, 1994; MR1446489]. For this reason, the paper goes far beyond the domain of symmetric real matrices and solves the corresponding problems for Hermitian or quaternionic matrices, as well as for the Lorentz cone and the exceptional Albert algebra of dimension 27. The reading of this remarkable and well-written paper is recommended to graduate students willing to learn techniques for symmetric random matrices.

Gérard Letac

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