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Blow-up solutions for fully nonlinear equations: existence, asymptotic estimates and uniqueness. (English summary)

The authors study the existence, asymptotic boundary estimates and uniqueness of solutions to the following problem:

\[
\begin{cases}
H(x, u, Du, D^2u) = f(u) + h(x) & \text{in } \Omega, \\
u = \infty & \text{on } \partial\Omega,
\end{cases}
\]

where \(\Omega\) is a bounded \(C^2\) domain in Euclidean space, \(H\) is a fully nonlinear uniformly elliptic operator of second order, \(f\) is a non-decreasing positive function in \((0, \infty)\), and \(h\) is a continuous function on \(\Omega\) with a certain boundary behavior.

The main contribution of this paper is to obtain properties of solutions to (1) for a quite general class of \(H\), \(f\), and \(h\). In particular, the authors treat cases when the coefficients of \(H\) are unbounded on \(\Omega\), or \(h\) is unbounded and sign-changing on \(\Omega\).

To deduce the main theorems, they elaborately use the method of sub- and super-solutions, comparison arguments, the Aleksandrov-Bakelman-Pucci estimates, the Keller-Osserman condition on \(f\), exhaustion of domains, and so on. Seunghyeok Kim

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