MR3935863 35J60 35B40 35B44 35J70
Mohammed, Ahmed (1-BLS); **Rădulescu, Vicenţiu D.** (SV-IMFM);
Vitolo, Antonio (I-SLRN-CVE)

Blow-up solutions for fully nonlinear equations: existence, asymptotic estimates and uniqueness. (English summary)

Adv. Nonlinear Anal. **9** (2020), *no. 1,* 39–64.

The authors study the existence, asymptotic boundary estimates and uniqueness of solutions to the following problem:

$$(1)\qquad \begin{cases} H(x,u,Du,D^2u)=f(u)+h(x) & \text{in } \Omega,\\ u=\infty & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded C^2 domain in Euclidean space, H is a fully nonlinear uniformly elliptic operator of second order, f is a non-decreasing positive function in $(0,\infty)$, and h is a continuous function on Ω with a certain boundary behavior.

The main contribution of this paper is to obtain properties of solutions to (1) for a quite general class of H, f, and h. In particular, the authors treat cases when the coefficients of H are unbounded on Ω, or h is unbounded and sign-changing on Ω.

To deduce the main theorems, they elaborately use the method of sub- and super-solutions, comparison arguments, the Aleksandrov-Bekel′man-Pucci estimates, the Keller-Osserman condition on f, exhaustion of domains, and so on. *Seunghyeok Kim*

References

1. N. Abatangelo, Very large solutions for the fractional Laplacian: Towards a fractional Keller–Osserman condition, *Adv. Nonlinear Anal.* **6** (2017), no. 4, 383–405. MR3719289
2. S. Alarcón and A. Quaas, Large viscosity solutions for some fully nonlinear equations, *NoDEA Nonlinear Differential Equations Appl.* **20** (2013), no. 4, 1453–1472. MR3078238
3. M. E. Amendola, L. Rossi and A. Vitolo, Harnack inequalities and ABP estimates for nonlinear second-order elliptic equations in unbounded domains, *Abstr. Appl. Anal.* **2008** (2008), Article ID 178534. MR2407278
4. A. Ancona, On strong barriers and an inequality of Hardy for domains in $\mathbf{R}^n$, *J. Lond. Math. Soc. (2)* **34** (1986), no. 2, 274–290. MR0856511
5. C. Bandle and M. Marcus, “Large” solutions of semilinear elliptic equations: Existence, uniqueness and asymptotic behaviour, *J. Anal. Math.* **58** (1992), 9–24. MR1226934
6. L. Caffarelli, M. G. Crandall, M. Kocan and A. Święch, On viscosity solutions of fully nonlinear equations with measurable ingredients, *Comm. Pure Appl. Math.* **49** (1996), no. 4, 365–397. MR1376656
7. L. A. Caffarelli, Interior a priori estimates for solutions of fully nonlinear equations, *Ann. of Math. (2)* **130** (1989), no. 1, 189–213. MR1005611
8. L. A. Caffarelli and X. Cabré, *Fully Nonlinear Elliptic Equations*, Amer. Math. Soc. Colloq. Publ. 43, American Mathematical Society, Providence, 1995. MR1351007
9. I. Capuzzo-Dolcetta, F. Leoni and A. Vitolo, The Alexandrov–Bakelman–Pucci weak maximum principle for fully nonlinear equations in unbounded domains,

Comm. Partial Differential Equations **30** (2005), no. 10–12, 1863–1881. MR2182315
10. O. Costin and L. Dupaigne, Boundary blow-up solutions in the unit ball: Asymptotics, uniqueness and symmetry, *J. Differential Equations* **249** (2010), no. 4, 931–964. MR2652158
11. O. Costin, L. Dupaigne and O. Goubet, Uniqueness of large solutions, *J. Math. Anal. Appl.* **395** (2012), no. 2, 806–812. MR2948269
12. M. G. Crandall, H. Ishii and P.-L. Lions, User's guide to viscosity solutions of second order partial differential equations, *Bull. Amer. Math. Soc. (N.S.)* **27** (1992), no. 1, 1–67. MR1118699
13. M. G. Crandall, M. Kocan, P. L. Lions and A. Święch, Existence results for boundary problems for uniformly elliptic and parabolic fully nonlinear equations, *Electron. J. Differential Equations* **1999** (1999), Paper No. 24. MR1696765
14. F. Da Lio and B. Sirakov, Symmetry results for viscosity solutions of fully nonlinear uniformly elliptic equations, *J. Eur. Math. Soc. (JEMS)* **9** (2007), no. 2, 317–330. MR2293958
15. G. Díaz and R. Letelier, Explosive solutions of quasilinear elliptic equations: Existence and uniqueness, *Nonlinear Anal.* **20** (1993), no. 2, 97–125. MR1200384
16. M. Dindoš, Large solutions for Yamabe and similar problems on domains in Riemannian manifolds, *Trans. Amer. Math. Soc.* **363** (2011), no. 10, 5131–5178. MR2813411
17. S. Dumont, L. Dupaigne, O. Goubet and V. Rădulescu, Back to the Keller–Osserman condition for boundary blow-up solutions, *Adv. Nonlinear Stud.* **7** (2007), no. 2, 271–298. MR2308040
18. L. Dupaigne, M. Ghergu and V. Rădulescu, Lane–Emden–Fowler equations with convection and singular potential, *J. Math. Pures Appl. (9)* **87** (2007), no. 6, 563–581. MR2335087
19. L. Escauriaza, $\mathbf{W}^{2,n}$ a priori estimates for solutions to fully nonlinear equations, *Indiana Univ. Math. J.* **42** (1993), no. 2, 413–423. MR1237053
20. M. J. Esteban, P. L. Felmer and A. Quaas, Superlinear elliptic equation for fully nonlinear operators without growth restrictions for the data, *Proc. Edinb. Math. Soc. (2)* **53** (2010), no. 1, 125–141. MR2579683
21. G. Galise and A. Vitolo, Viscosity solutions of uniformly elliptic equations without boundary and growth conditions at infinity, *Int. J. Differ. Equ.* **2011** (2011), Article ID 453727. MR2854945
22. J. García-Melián, Uniqueness of positive solutions for a boundary blow-up problem, *J. Math. Anal. Appl.* **360** (2009), no. 2, 530–536. MR2561251
23. J. García-Melián, Large solutions for an elliptic equation with a nonhomogeneous term, *J. Math. Anal. Appl.* **434** (2016), no. 1, 872–881. MR3404590
24. M. Ghergu and V. D. Rădulescu, *Singular Elliptic Problems: Bifurcation and Asymptotic Analysis*, Oxford Lecture Ser. Math. Appl. 37, Clarendon Press, Oxford, 2008. MR2488149
25. D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Grundlehren Math. Wiss. 224, Springer, Berlin, 1977. MR0473443
26. F. Gladiali and G. Porru, Estimates for explosive solutions to p-Laplace equations, in: *Progress in Partial Differential Equations. Vol. 1* (Pont-à-Mousson 1997), Pitman Res. Notes Math. Ser. 383, Longman, Harlow (1998), 117–127. MR1628068
27. J. A. Goldstein, An appreciation of my teacher, M. M. Rao, in: *Stochastic Processes and Functional Analysis*, Lecture Notes Pure Appl. Math. 238, Dekker, New York (2004), 3–5. MR2059892
28. J. B. Keller, On solutions of $\Delta u = f(u)$, *Comm. Pure Appl. Math.* **10** (1957), 503–510. MR0091407

29. S. Koike and A. Święch, Weak Harnack inequality for fully nonlinear uniformly elliptic PDE with unbounded ingredients, *J. Math. Soc. Japan* **61** (2009), no. 3, 723–755. MR2552914
30. J. López-Gómez and L. Maire, Uniqueness of large positive solutions, *Z. Angew. Math. Phys.* **68** (2017), no. 4, Article ID 86. MR3672379
31. M. Marcus and L. Véron, Uniqueness and asymptotic behavior of solutions with boundary blow-up for a class of nonlinear elliptic equations, *Ann. Inst. H. Poincaré Anal. Non Linéaire* **14** (1997), no. 2, 237–274. MR1441394
32. M. Marcus and L. Véron, The boundary trace of positive solutions of semilinear elliptic equations: The subcritical case, *Arch. Ration. Mech. Anal.* **144** (1998), no. 3, 201–231. MR1658392
33. M. Marcus and L. Véron, The boundary trace and generalized boundary value problem for semilinear elliptic equations with coercive absorption, *Comm. Pure Appl. Math.* **56** (2003), no. 6, 689–731. MR1959738
34. J. H. Michael, A general theory for linear elliptic partial differential equations, *J. Differential Equations* **23** (1977), no. 1, 1–29. MR0425346
35. A. Mohammed and G. Porru, Harnack inequality for non-divergence structure semilinear elliptic equations, *Adv. Nonlinear Anal.* (2016), DOI 10.1515/anona-2016-0050. MR3836114
36. A. Mohammed and G. Porru, Large solutions for non-divergence structure equations with singular lower order terms, *Nonlinear Anal. Real World Appl.* **35** (2017), 470–482. MR3595337
37. A. Mohammed and G. Porru, Large solutions to non-divergence structure semilinear elliptic equations with inhomogeneous term, *Adv. Nonlinear Anal.* (2017), DOI 10.1515/anona-2017-0065. MR3918389
38. A. Mohammed, G. Porru and A. Vitolo, Harnack inequality for nonlinear elliptic equations with strong absorption, *J. Differential Equations* **263** (2017), no. 10, 6821–6843. MR3693190
39. A. Mohammed and A. Vitolo, Large solutions of fully nonlinear equations: Existence and uniqueness, preprint. MR4016517
40. R. Osserman, On the inequality $\Delta u \geq f(u)$, *Pacific J. Math.* **7** (1957), 1641–1647. MR0098239
41. V. D. Rădulescu, Singular phenomena in nonlinear elliptic problems: From blow-up boundary solutions to equations with singular nonlinearities, Handbook of Differential Equations: Stationary Partial Differential Equations. Vol. IV, Handb. Differ. Equ., Elsevier, Amsterdam (2007), 485–593. MR2569336
42. D. Repovš, Asymptotics for singular solutions of quasilinear elliptic equations with an absorption term, *J. Math. Anal. Appl.* **395** (2012), no. 1, 78–85. MR2943604
43. A. Święch, $\mathbf{W}^{1,p}$-interior estimates for solutions of fully nonlinear, uniformly elliptic equations, *Adv. Differential Equations* **2** (1997), no. 6, 1005–1027. MR1606359
44. N. S. Trudinger, Comparison principles and pointwise estimates for viscosity solutions of nonlinear elliptic equations, *Rev. Mat. Iberoam.* **4** (1988), no. 3–4, 453–468. MR1048584
45. L. Véron, Semilinear elliptic equations with uniform blow-up on the boundary, *J. Anal. Math.* **59** (1992), 231–250. MR1226963
46. A. Vitolo, M. E. Amendola and G. Galise, On the uniqueness of blow-up solutions of fully nonlinear elliptic equations, *Discrete Contin. Dyn. Syst.* **2013** (2013), 771–780. MR3462421
47. Z. Zhang, A boundary blow-up elliptic problem with an inhomogeneous term, *Nonlinear Anal.* **68** (2008), no. 11, 3428–3438. MR2401356

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.