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★**Muchnik degrees and Medvedev degrees of randomness notions. (English summary)**

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Let CR, SR, KLR, and MLR be the classes of computably random, Schnorr random, Kolmogorov-Loveland random, and Martin-Löf random reals, respectively.

Let $\le_s$ denote the uniform (strong) reducibility of mass problems known as Medvedev reducibility, and let $\le_w$ denote the non-uniform (weak) version known as Muchnik reducibility.

While it was shown by A. O. Nies, F. C. Stephan and S. A. Terwijn [J. Symbolic Logic **70** (2005), no. 2, 515–535; MR2140044] that CR $\le_w$ SR, the author obtains an interesting counterpoint by showing as his main theorem that CR $\not\le_s$ SR.

Analogously, it was shown by W. Merkle et al. [Ann. Pure Appl. Logic **138** (2006), no. 1-3, 183–210; MR2183813] that MLR $\le_w$ KLR; the author poses an open problem at the end: Is MLR $\le_s$ KLR?

The reviewer is pleased to give an affirmative answer as follows. Given a KL-random set $A = A_0 \oplus A_1$, we output bits of either A_0 or A_1, switching whenever we notice that the smallest possible randomness deficiency (c such that $\forall n\,(K(A_i \upharpoonright n) \ge n - c)$) increases. Since by [W. Merkle et al., op. cit.] one of A_0, A_1 is ML-random, switching will occur only finitely often. Thus our output will have an infinite tail that is ML-random, and hence be itself ML-random.

{For the collection containing this paper see MR3890085}

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