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★**Exact solutions in three-dimensional gravity.**
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When I was asked to review this book, the first thing that popped into my mind was the classic Cambridge monograph on mathematical physics, *Exact solutions of Einstein's field equations* [H. Stephani et al., second edition, Cambridge Monogr. Math. Phys., Cambridge Univ. Press, Cambridge, 2003; MR2003646; D. Kramer et al., *Exact solutions of Einstein's field equations*, Cambridge Univ. Press, Cambridge, 1980; MR0614593]. Thus, I was not surprised at all when in its preface the author, Alberto A. García-Díaz, acknowledges the catalogue by Stephani, Kramer, MacCallum, Hoenselaers, and Herlt as his inspiring muse. Going through *Exact solutions in three-dimensional gravity*, I received more and more confirmations of my first feeling because this new Cambridge monograph is written in the same neat and concise style as its four-dimensional counterpart. The subject covered in this treatise is organized as well following the same logical exposition adopted in the book by Stephani et al., that is, classifying the known solutions from the point of view both of the geometrical symmetries of the manifold (like spherical, axial, cyclic) and of the matter content supporting the spacetime (the cases presented correspond to solutions for point particles, perfect fluid stars and cosmological models, black holes, dilaton and inflaton, electromagnetics and stringy metrics). A relevant part of this book (one hundred pages over the total 430) is devoted to the topologically massive gravity theory focusing on the inhomogeneous Bianchi models, the Kundt solution, and the Cotton N waves arising in that framework.

In Chapter 1, after writing down the field equations for gravity in 3D and briefly commenting about the main differences with respect to general relativity (like the absence of gravitational waves in 3D, of geodesic deviation in dust spacetimes, and of black holes in asymptotically flat spacetimes), the author continues with the algebraic classification of the Cotton (this particular topic will be deepened again in the ending Chapter 20, which enlightens the Bianchi identities and the criteria for conformal flatness) and energy-momentum tensors. Similarly, Chapter 16 provides the field equations for the topologically massive gravity which follows from the Chern-Simons action (without deriving them). Then, the reader can peruse chapter after chapter to find a schematic but comprehensive list of the nowadays available solutions for the field equations accounting for 3D gravity.

Most of the sections of this book begin with the generic (from a geometrical perspective) line-element under analysis (for example writing $ds^2 = -A(x,y)^2dt^2 + e^{2\eta(x,y)}(dx^2+dy^2)$ for a static cyclic symmetric universe in isotropic coordinates) leaving the explicit expression for the components of the metric tensor unspecified. Next, a possible transformation of the coordinate system for simplifying the mathematical problem of solving the field equations can be mentioned. Then, the components of the Einstein tensor for the chosen geometry are provided as a system of differential equations for the functions entering the line-element. The last step consists in exhibiting the formal integral construction of the solution and in providing an analytical result in closed form, when possible, avoiding displaying all the intermediate mathematical manipulations which led to the final solution. As a consequence, the essential physical features of the solution, like the dependence of the energy density or of the electromagnetic field on the coordinates, are accounted for. However, the physical discussion is kept to a minimum level, and all the subjects like the thermodynamical and horizon properties of black holes, and Buchdahl's limit for the mass of a star in hydrodynamic equilibrium, are just handled in a couple of pages, each referring the reader to the original research papers for details (this book provides about 180 references to the original literature on which it is based).

The majority of the explicit spacetime metric tensors reviewed in this book can be written in terms of just a handful of sets of elementary functions, like polynomials, and exponential and/or trigonometric functions. Therefore, according to the reviewer, as a first example, colleagues working on the construction of a quantum theory of gravity may find this manual a valuable resource (in particular Chapter 12, which deals with black holes coupled to nonlinear electrodynamics) from which they can read off a certain spacetime metric tensor and try to build their quantum proposal on it, benefiting from the opportunity of facing fewer mathematical difficulties than working with the 4D Einstein theory. On the other hand, exact solutions of systems of coupled nonlinear partial differential equations are scarce. Hence, mathematicians whose interests lie in the generating technique subject can extract directly from this text the differential field equations for each type of metric (ignoring how they were formulated in the first instance) and investigate whether they admit some type of mathematical reduction which in turn can help in finding new exact solutions (as has already happened thanks to the Bäcklund, Geroch, and Wainwright transformations in 4D general relativity, and is briefly discussed here for the case of the $SL(2,R)$ transformation for dilatonic spacetimes). Analogously, exact solutions supported by fluids involving stresses or heat flows are not mentioned in this essay, calling for a study about their existence, and/or for an accompanying textbook which reviews them as well. Last but not least, in this monograph the cosmological Friedmann models and the star solutions displayed are compared and contrasted with those arising in 4D gravity, providing a smooth transition from general relativity for the neophytes of lower-dimensional theories.

To conclude, *Exact solutions in three-dimensional gravity* by Alberto A. García-Díaz can serve as both a reliable and an authoritative reference for those who are already familiar with this topic and need to check the form of a certain solution before applying it, and for those who are approaching lower-dimensional theories for the first time and would just like to get a taste of what 3D gravity looks like. *Daniele Gregoris*