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Casimir squared correction to the standard rotator Hamiltonian for the O(n) sigma-model in the delta-regime. (English summary)
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In a previous paper [J. High Energy Phys. **2017**, no. 6, 150; MR3681875], the authors found that the isospin susceptibility of the O(n) sigma model calculated in the standard rotator Hamiltonian approximation differs from the next-to-next-to leading order (NNLO) chiral perturbation theory (χPT) result in terms vanishing like $1/\ell$, for $\ell = L_t/L$. Here L_t is the length of the time interval, and L is the length of the space interval. Furthermore they showed that this deviation, which can be described by a correction to the rotator spectrum, is proportional to the square of the quadratic Casimir invariant, and occurs at 3-loops. The proportionality constant is determined by the NNLO LEC's (low energy constants) of χPT.

In the paper under review, in $d=2$ space-time dimensions, the authors compare this expectation with analytic non-perturbative results on the spectrum, for $n=3,4$, by J. Balog and Á. Hegedűs [J. Phys. A **37** (2004), no. 5, 1881–1901; MR2044197], and for $n=4$, by N. A. Gromov, V. A. Kazakov and P. G. Vieira [J. High Energy Phys. **2009**, no. 12, 060; MR2592995], respectively. In Section 3 of the paper under review, the authors find good agreement in both scenarios, i.e., in the $d=2$, O(3) and O(4) cases. They also generate more data than in the cited paper by Gromov, Kazakov and Vieira. The methods used to solve the TBA (thermodynamic Bethe ansatz) equations are described well in Appendix C. In Section 2, some preliminary results about the isospin susceptibility in χPT are presented. To this purpose, dimensional regularization techniques, 1-loop regularized sums over momenta, and 2-loop vacuum massless sunset diagrams are employed. In Section 4, the authors compute the effect for the $d=3$ O(n) model, but they have not yet found good data with which a comparison can be made. This difficulty is caused by the lack of a rigorous derivation of the TBA equations (or even of the Zamolodchikov S-matrix) from first principles starting with the 2d O(n) model in QFT (except for $n=4$, which is also a principal chiral model). Hence, the agreement of the results for $d=2$ provides extra evidence for the validity of both scenarios, and furthermore, it encourages the application of similar assumptions for $d>2$.

Several physical systems, like the Goldstone modes in the delta-regime of QCD (quantum chromodynamics), spontaneous symmetry breaking in condensed matter physics in $d=3$, and non-linear sigma models in $d=2$ are described by a quantum rotator to leading order. In all such systems the lowest energy momentum zero states of isospin I have to leading order, χPT energies with Casimir scaling of the form $E_I \propto C_{n,I}$, where $C_{n,I} = I(I+n-2)$ is the eigenvalue of the quadratic Casimir correction for isospin I. At the 1-loop level it turns out that this Casimir scaling still holds, but of course it is to be expected that at some higher order the standard rotator spectrum will have to be modified. The standard rotator describes a system where the length of the total magnetization on a time slice does not change in time. This is obviously not true for the full effective model given by χPT. As the authors notice, the actual deviation from the Casimir scaling was probably first observed by Balog and Hegedűs in their computation of the spectrum of the $d = 2$ O(3) non-linear sigma model in a small periodic box (circle) using the TBA. Of course, a deviation from the standard rotator spectrum can be established by explicit perturbative computations, as performed by the authors.

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