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Citations From References: 1224 From Reviews: 43

MR2354493 (2009k:35096) 35J70
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An extension problem related to the fractional Laplacian. (English summary)

Comm. Partial Differential Equations **32** (2007), *no. 7-9,* 1245–1260.

In the literature, the fractional Laplacian of order $s \in (0,1)$ is defined, up to constants, as

$$-\Delta^s v = \text{p.v.} \int_{\mathbb{R}^N} \frac{v(x)-v(y)}{|x-y|^{N+2s}}\, dy.$$

Now, let v be a smooth function defined on $\mathbb{R}^N$, and let

$$a = 1-2s.$$

Consider the following extension problem:

$$\begin{cases} -\text{div}(y^a \nabla u(x,y)) = 0 & \text{in } \mathbb{R}^N \times \mathbb{R}^+, \\ u(x,0) = v(x) & \text{on } \mathbb{R}^N. \end{cases} \tag{1}$$

In the paper under review, the authors start by proving a result which relates a nonlocal problem associated to the fractional power of the Laplacian to the local degenerate elliptic problem (1). Namely, they prove that there exists a constant C, which only depends on N and s, such that

$$\lim_{y \to 0^+} y^a u_y(x,y) = -C\Delta^s v.$$

This characterization then allows the authors to prove several regularity results concerning the solutions of problems involving the fractional Laplacian by exploiting purely local techniques related to problem (1). In particular, they prove both a Harnack and a boundary Harnack inequality, as well as a monotonicity formula for the energy in the same spirit of the local Almgren frequency formula. *Francesco Petitta*

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