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Podlubny, Igor (SK-KTU)
★**Fractional differential equations.**
An introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications.
Mathematics in Science and Engineering, 198.
Academic Press, Inc., San Diego, CA, 1999. *xxiv*+340 *pp.* $69.95. *ISBN* 0-12-558840-2

The field of mathematical analysis known as fractional calculus deals with the investigation and applications of derivatives and integrals of fractional (real and complex) order. Classical and modern results in this theory and its applications to integral and differential equations were presented in the books by S. G. Samko, the reviewer and O. I. Marichev [*Integrals and derivatives of fractional order and some of their applications* (Russian), "Nauka i Tekhnika", Minsk, 1987; MR0915556; *Fractional integrals and derivatives*, Translated from the 1987 Russian original, Gordon and Breach, Yverdon, 1993; MR1347689]. Many other previous publications have been devoted to the problems of fractional calculus and its applications.

The book under review continues work in this field. It consists of ten chapters of varying sizes and an appendix. Chapter 1 (39 pages) contains some known results from the theory of gamma- and beta-functions, and the Mittag-Leffler and Wright functions. Chapter 2 (79 pages), comprising a quarter of the book, is devoted to the Grunwald-Letnikov and Riemann-Liouville fractional integrals and derivatives and some of their properties. Existence and uniqueness theorems for ordinary linear and nonlinear fractional differential equations are studied in Chapter 3 (16 pages). Chapters 4 (11 pages) and 5 (10 pages) deal with the solution of fractional differential equations with constant coefficients using the Laplace transform and the fractional Green function, respectively. In Chapter 6 (40 pages), the Mellin transform and power series approaches are illustrated by solving examples of fractional differential equations, while the Babenko symbolic method and the method of orthogonal polynomials are illustrated by examples of simple singular integral equations with power and Cauchy kernels. A numerical treatment of the fractional derivative based on its approximation by a finite difference is considered in Chapter 7 (23 pages), and this numerical approach is illustrated in four examples in Chapter 8 (20 pages). Chapter 9 (18 pages) deals with fractional-order systems and controllers, and Chapter 10 (46 pages) contains some applications of fractional calculus to problems in physics, mechanics, chemistry and biology. Tables of fractional derivatives are given in the appendix (3 pages).

{Reviewer's remarks: (1) The main title, "Fractional differential equations", does not correspond to the contents of the book, which is primarily devoted to fractional integrals and derivatives and some of their applications.

{(2) The results presented on the theory of fractional derivatives and integrals, in particular on the theory of fractional differential equations, are basically well known. Most of the proofs are formal and some of them contain mistakes, for example, the proof of uniqueness in Theorem 3.4 and the solution of Example 6.1.

{(3) The author does not use correct appellations. The fractional derivative which he calls the Caputo derivative is a particular case of the Riemann-Liouville fractional derivative. He indicates (page 88) that M. M. Dzhrbashyan and A. Nersesyan first considered and used compositions of several fractional derivatives, but calls these constructions the

Miller-Ross fractional derivatives. K. S. Miller and B. Ross [*An introduction to the fractional calculus and fractional differential equations*, Wiley, New York, 1993; MR1219954] considered only a particular case of such constructions and called them sequential fractional derivatives. Particular cases of other well-known special functions are also referred to incorrectly: the Rabotnov and Miller-Ross functions for the Mittag-Leffler function, and the Mainardi function for the Wright function.} *Anatoly Kilbas*