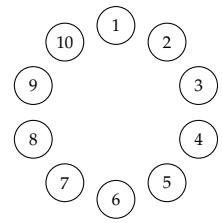
February 7, 2019 Name:_____

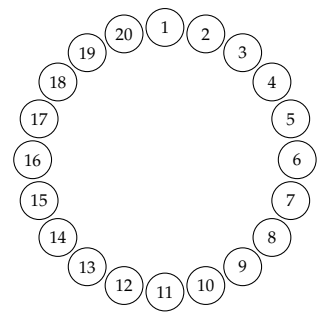
M245: Introductory Activity

Imagine we start with n people numbered 1 to n around a circle. Starting by eliminating person 2, we eliminate every *second* remaining person until only one remains.

1. Perform the scenario above when there are 10 people (i.e. when $n = 10$). See the side note for a diagram.



2. Run the scenario when there are 20 people (i.e. when $n = 20$). See the side note for a diagram.



3. The number of the remaining survivor is uniquely determined by how many people we start with. We call this number $J(n)$. Write a pseudo-code that would compute $J(n)$ given an input of n.

4. Now, in your group, find $J(n)$ for each n from 1 to 20 (remember we have already found $J(10)$ and $J(20)$).

How sure are you that your code will work correctly?

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$J(n)$																				

5. Do you see any patterns in the chart above? Do see how we could exploit those patterns? Can you determine $J(10000000)$?

If you have a computer and can quickly implement your pseudo-code you can use that instead.
How long would your computer program take to compute $J(10000000)$?

A light switch game has n switches; all initially begin in the OFF position. The first switch may always be flipped regardless of the other switches. In order to flip switch i (either way), the following must happen:

i. Switch $(i-1)$ must be ON and

ii. all earlier switches must be OFF.

The goal is to get the last switch ON while all others are OFF (see Figure 1).

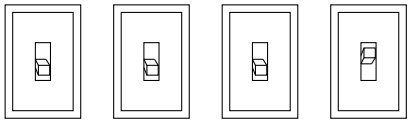


Figure 1: The goal of the light switch game with four switches.

1. Find the minimal number of moves required to win the game with 4 switches (pictured in Figure 1.

2. Play the games with other numbers of switches. Do you see any patterns? Can you guarantee any of those patterns hold forever?

It might be helpful to make a chart as we did on the previous page.

3. What is the minimal number of moves required to win the game with 50000 switches?