Name: _____ March 03, 2008

Derivatives of exponential functions

The setup

So far, we've learned rules for differentiating polynomials, trig functions, logarithmic functions, compositions of functions, and any product or quotient of the above.

In this lab, we'll explore the derivative of *exponential functions,* in particular, the function,

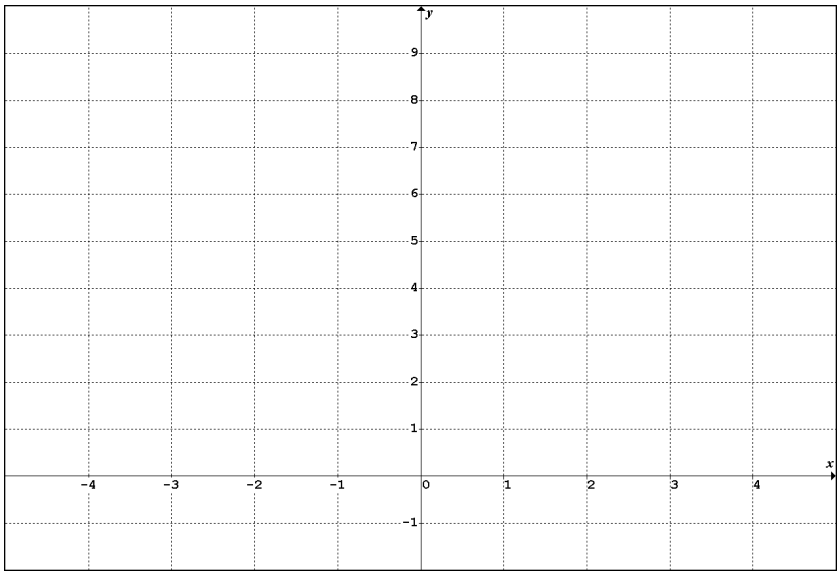
$$f(x) = 2^x$$

Step 1: Graphical analysis

First we'll explore the derivative of our function graphically.

1. Enter the function as **y1** and the numerical derivative (`nDeriv`) as **y2** on your calculator.

2. Graph the function and its numerical derivative (hint: use *thick style* for the derivative so you can distinguish the graphs). Sketch and label the graphs below:



3. Explain *in complete sentences* how the graphs are related.

Step 2: Numerical analysis

Now we'll explore the numerical relationship between the function and its derivative.

4. Set **y3 =** $\frac{\text{y2}}{\text{y1}}$. Create a table of values and explain what you can learn from the table.

Step 3: Algebraic analysis

Finally, we'll explore the relationship algebraically.

5. Explain why $f'(x) = \lim_{h \to 0} \frac{2^{x+h} - 2^x}{h}$.

6. Using the equation above as a starting point, show that $f'(x) = 2^x \lim_{h \to 0} \frac{2^h - 1}{h}$.

7. Find $\lim_{h \to 0} \frac{2^h - 1}{h}$. Does this match with the numerical and graphical evidence that you've gathered?

Put it all together

8. Make a conjecture: If $f(x) = 2^x$, then $f'(x) =$ _____