

## TIMSS fraction item

TIMSS is an international set of tests on mathematics and science which is given every four years in grades 4 and 8 to a sample of students, and occasionally for a sample of students taking advanced mathematics and physics in their last year in high school. All of these will be given in 2015.

The following useful link gives access to the released TIMSS-2011 items and the scores different countries made on these items.

<http://timssandpirls.bc.edu/timss2011/international-released-items.html>

Here is one eighth grade item on fractions.

### Item M 052228

Which shows a correct method for finding  $1/3 - 1/4$ ?

A  $(1 - 1) / (4 - 3)$

B  $1 / (4 - 3)$

C  $(3 - 4) / (3 * 4)$

D  $(4 - 3) / (3 * 4)$

Here are some results. A second link is needed to see what percent students had on various answers, both correct and incorrect. Here is the link.

<http://timss.bc.edu/timss2011/international-database.html>

and then click on almanac.

The numbers are percents.

|           | Correct | A    | B    | C    | D    |
|-----------|---------|------|------|------|------|
| Average   | 37.1    | 25.4 | 26.0 | 9.4  | 37.1 |
| Korea     | 86.0    | 2.7  | 6.9  | 4.2  | 86.0 |
| Singapore | 83.1    | 4.8  | 5.5  | 6.5  | 83.1 |
| Taipei    | 82.0    | 2.9  | 7.7  | 7.0  | 82.0 |
| Hong Kong | 77.0    | 4.0  | 8.7  | 10.0 | 77.0 |
| Japan     | 65.3    | 15.4 | 11.1 | 8.2  | 65.3 |
| Russia    | 62.8    | 12.3 | 18.8 | 4.8  | 62.8 |

|           |      |      |      |      |      |
|-----------|------|------|------|------|------|
| Average   | 37.1 | 25.4 | 26.0 | 9.4  | 37.1 |
| Australia | 33.8 | 30.2 | 21.9 | 12.8 | 33.8 |
| US        | 29.1 | 32.5 | 26.1 | 10.7 | 29.1 |
| England   | 28.2 | 24.5 | 32.8 | 11.0 | 28.2 |
| Finland   | 16.1 | 42.3 | 29.5 | 8.7  | 16.1 |

Canada did not take TIMSS as a country but some provinces did as did some US states. Here are some of these results

|         |      |      |      |      |      |
|---------|------|------|------|------|------|
| Mass.   | 44.4 | 21.4 | 20.8 | 9.9  | 44.4 |
| Calif.  | 38.0 | 28.2 | 21.6 | 11.0 | 38.0 |
| Minn.   | 35.1 | 23.5 | 26.3 | 14.0 | 35.1 |
| Quebec  | 33.0 | 27.3 | 23.0 | 13.0 | 33.0 |
| Ontario | 32.5 | 27.7 | 22.4 | 14.0 | 32.5 |
| Conn.   | 31.3 | 21.8 | 25.8 | 17.7 | 31.3 |
| Alberta | 27.8 | 34.7 | 23.7 | 12.3 | 27.8 |

There was a small percent of students who left the answer blank and an even smaller percent who did not reach this question.

One interesting fact is that among the 42 countries which tested 8th grade students, Finland had the highest percent of students who picked answer A and the third lowest percent correct. Chile had 11.7 correct and Sweden had 14.4 percent correct. The Finnish result is likely a surprise to the people who have praised the Finnish school system for their results on another international test, PISA. However university and technical college mathematics faculty in Finland will not be surprised. See an article signed by over 200 of them which is on the web at

<http://solmu.math.helsinki.fi/2005/erik/PisaEng.html>

Students who pick answers A or B have no idea what fractions really are. The answer  $\frac{1}{3} - \frac{1}{4} = \frac{(1-1)}{(4-3)}$  cannot be right since  $1-1$  is 0 so the fraction on the right is 0 and  $\frac{1}{3} - \frac{1}{4}$  is not zero. It cannot be  $\frac{1}{(4-3)}$  which is  $\frac{1}{1} = 1$  and  $\frac{1}{3}$  is less than 1 and something has been subtracted from it. For C,  $\frac{1}{3} > \frac{1}{4}$  so  $\frac{1}{3} - \frac{1}{4} > 0$ , but  $\frac{(3-4)}{(3*4)} = \frac{-1}{(3*4)} < 0$  so C is not correct. There is a little reason why students who knew something about fractions might have picked C. One can be careless and get a minus sign wrong. However, there is no good excuse for picking A or B.

Among the 42 countries which tested 8th grade students, the US had almost 60% picking A or B and Finland had over 70%.

Part of the Common Core treatment of fractions is to help students learn that fractions are numbers and they can and should be thought of as living on a number line. Addition and subtraction of whole numbers has been done and students are moderately good at knowing what is happening there. Fraction addition and subtraction should be thought of as coming from whole number addition and subtraction, with equivalent fractions being the new part which makes this possible. For a short summary on how to do this see a paper in *American Educator* by Hung-Hsi Wu.

<http://www.aft.org/pdfs/americaneducator/fall2011/wu.pdf>

Lest a reader think that only East Asian countries can teach fractions well, one can consider the Flemish part of Belgium, i.e. Flanders. They did not take TIMSS-2011, but took TIMSS-1995. There was a somewhat similar fraction problem, to add three fractions. The US had a higher percent adding numerators and denominators than the international average and a lower percent getting the correct answer than the international average. We had more adding numerators and denominators than got the correct answer. Flanders had a very small percent adding numerators and denominators, less than some of the well performing East Asian countries, and a very high percent getting the result correct, again higher than some of these East Asian countries.

What content is taught and how it is taught from a mathematical point of view is very important. One cannot just assume that things like this will be done adequately, much less done well.

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